Sydney Girls High School

2007
TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION

Mathematics

Extension 2

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2007 HSC Examination Paper in this subject.

General Instructions

- **Reading Time – 5 minutes**
- **Working time – 3 hours**
- **Attempt ALL questions**
- **ALL questions are of equal value**
- **All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.**
- **Standard integrals are supplied**
- **Board-approved calculators may be used**
- **Diagrams are not to scale**
- **Each question attempted should be started on a new sheet. Write on one side of the paper only.**

Question One (15 marks) **Marks**

a) i) Show that $\sin(A-B)+\sin(A+B)=2\sin A\cos B$ 1

ii) Hence or otherwise find $\int \sin 3x\cos x\,dx$ 1

b) Find $\int x\sqrt{x^2+7}\,dx$ 2

c) By completing the square find $\int \frac{dx}{\sqrt{x^2-4x+8}}$ 2

d) i) Given that $\frac{4x^2+3x+33}{(x^2+16)(x+1)} \equiv \frac{Ax+B}{(x^2+16)}+\frac{C}{(x+1)}$ where A, B and C are real numbers, find A, B and C 3

ii) Hence find $\int \frac{4x^2+3x+33}{(x^2+16)(x+1)}dx$ 3

e) Find $\int \frac{4dx}{x^2\sqrt{x^2-4}}$ 3

Question Two (15 marks) **Marks**

a) Given $z_1 = 2-i$ and $z_2 = 3+4i$ express z_1z_2 in the form $a+bi$ 1

b) i) Express $\frac{1-3i}{1+2i}$ in modulus argument form. 3

ii) Hence find $\left(\frac{1-3i}{1+2i}\right)^7$ in simplest modulus argument form 1

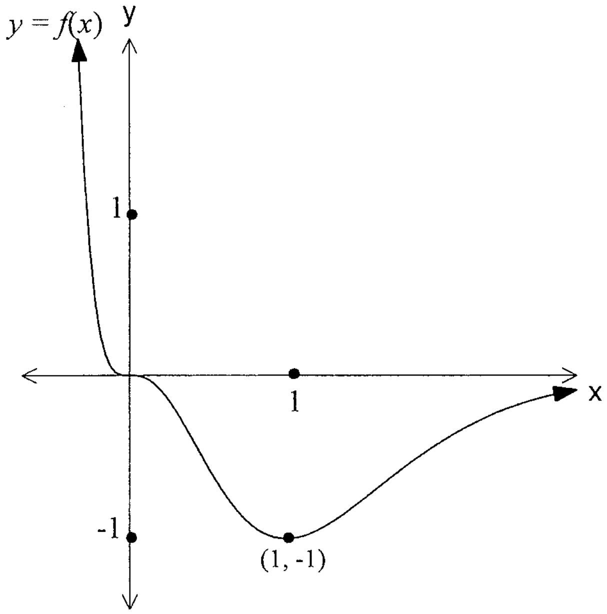
c) i) Find the square roots of $-15-8i$ in the form $a+bi$ 2

ii) Hence solve $z^2+(2i-3)z+(5-i)=0$ 2

d) Sketch the region represented by $0 \le \text{Re}(z^2) \le 4$ on an Argand Diagram 3

e) The equation $|z+2|+|z-6|=10$ represents an ellipse in the Argand diagram. Sketch the ellipse, and clearly showing the centre and the lengths of the minor and major axes 3

Question Three (15 marks) **Marks**



a) The diagram above shows the graph of $y = f(x)$. The graph has a minimum turning point at (1, -1), a horizontal point of inflexion at the origin and is asymptotic to the positive X - axis

Draw separate 1/3 page sketches of the graphs of the following showing relevant features:

i) $y = f(-x)$ 1

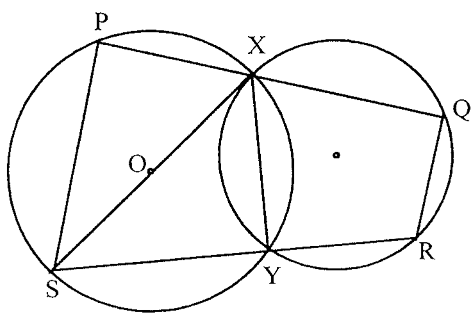
ii) $y = |f(x)|$ 2

iii) $y^2 = f(x)$ 2

iv) $y = \dfrac{f(x)}{x}$ 2

v) $y = \dfrac{d}{dx}[f(x)]$ 2

b) In the diagram below, SX is a diameter of the circle centre O



i) Prove that PQ $\perp$ QR 2

ii) Prove $\angle PYQ = \angle SXR$ 4

Question Four (15 marks) **Marks**

a) Given the hyperbola $\frac{y^2}{16}-\frac{x^2}{9}=1$

i) Determine the eccentricity 1

ii) Find the co ordinates of the foci 1

iii) Determine the equations of the directrices 1

iv) Determine the equations of the asymptotes 1

v) Sketch the hyperbola 1

b) i) Show that the point P with co ordinates $\left(2\sec\theta,\ \sqrt{5}\tan\theta\right)$ lies on the hyperbola $\frac{x^2}{4}-\frac{y^2}{5}=1$ 1

ii) From the equation of the hyperbola $\frac{x^2}{4}-\frac{y^2}{5}=1$ derive the equation of the tangent at the point $P\left(2\sec\theta,\ \sqrt{5}\tan\theta\right)$ 3

iii) The tangent cuts the asymptotes at the points A and B. Find the co ordinates of A and B 4

iv) Show that P is the midpoint of AB 2

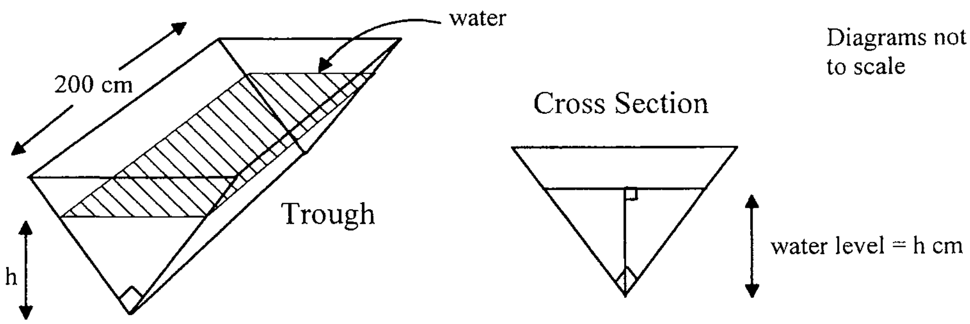
Question Five (15 marks) **Mark**

a) Without using calculus draw a 1/3 page sketch of the graph of $y = \dfrac{1}{x^2 + x - 6}$ 3

b) Find the equation of the tangent to the curve $x^2 + y^2 + xy - 4 = 0$ at the point (0, 2) 3

c) The polynomial $P(x) = x^4 - 4x^3 + 11x^2 - 14x + 10$ has roots $a + ib$, $a + 2ib$ where a and b are real. Find the values of a and b 3

d) The cross section of a water trough is in the shape of a right isosceles triangle. The trough is 200 centimetres long.



Water is flowing into the trough at the rate of $12 cm^3 s^{-1}$.
Find the rate of change of the upper surface area of the water when the height of the water is 12cm 6

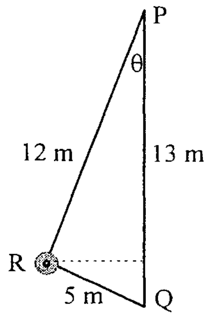
Question Six (15 marks) **Marks**

a) If α, β, γ are the roots of the equation $x^3 + 6x^2 + 5x + 5 = 0$ find the equation with roots:

i) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ 2

ii) $\frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\alpha\gamma}$ 2

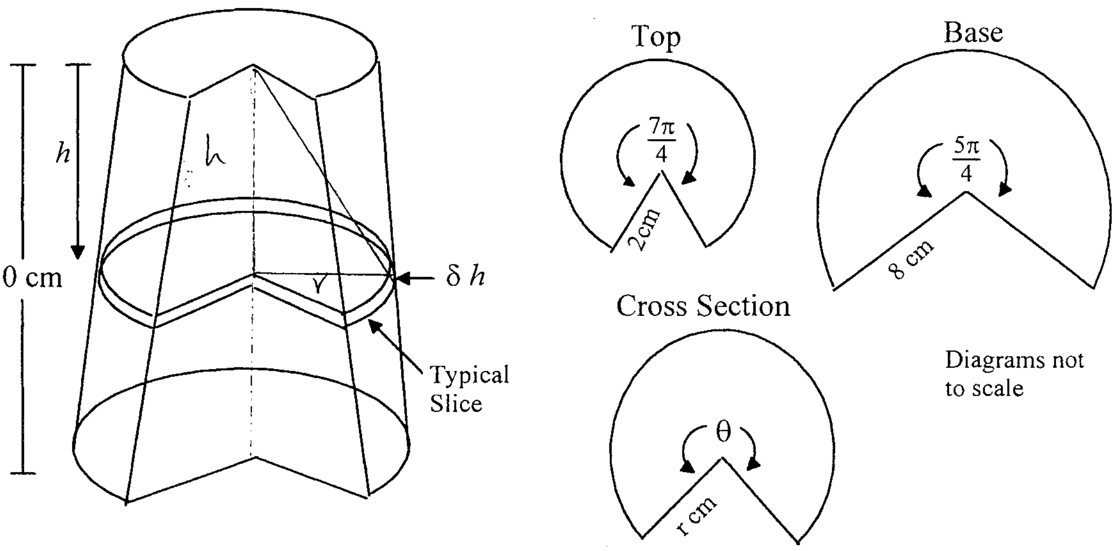
b) A 4kg mass at R rotates about PQ. How fast must the mass rotate (in metres per second) if the tension T_1 in the string PR is to be equal to the tension T_2 in the string QR? 5



Note that PQ is vertical

$g = 10ms^{-2}$

c) A solid has a base and top in the shape of a sector of a circle as shown below. The height of the solid is 20 cm. All other dimensions are shown on the right below. Note that angles are given in radians



Cross sections are taken perpendicular to the height. A typical slice is shown h centimetres from the <u>top</u> of the solid. A linear relationship exists between r and h, θ and h,

i) Find r in terms of h and θ in terms of h 3

ii) Hence find the volume of the solid 3

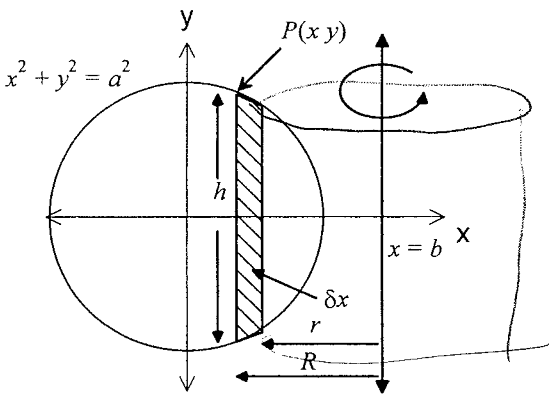
Question Seven (15 marks) **Marks**

a) Given that $P(x) = 3x^3 - 11x^2 + 8x + 4$ has a double root, fully factorise $P(x)$ 3

b) i) Use De Moivre's show $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ 3

ii) Hence solve $\sin 5x = \sin x$ for $0 \le x \le 2\pi$ 3

c) The diagram below show the graphs of $x^2 + y^2 = a^2$ and the line $x = b$ where $b > a$



i) The shaded strip of width δx and height h is rotated about the line $x = b$ to form a cylindrical shell. Find expressions for R, r and h and hence the volume of the shell formed. 3

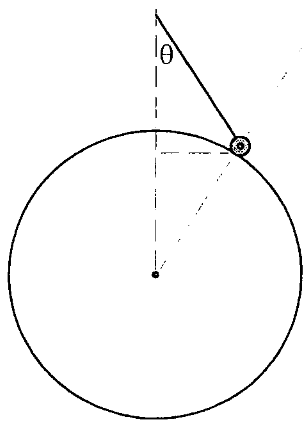
ii) Hence find the volume of the solid formed when when $x^2 + y^2 = a^2$ is rotated about the line $x = b$ 3

Question Eight (15 marks) **Marks**

a) Use Mathematical Induction to prove the following 4

$\cos(x+n\pi)=(-1)^n\cos x$ where $n>0$

b) From a point 1.3 metres above a sphere of radius 1.7 metres a mass of 6 kilograms is on the end of a string of length 1.7 metres. The mass moves in a horizontal circle of radius 0.8 metres with an angular velocity of $2\,rad.s^{-1}$



$g = 10ms^{-2}$

i) Copy the diagram and show all forces 2

ii) Find the tension in the string 3

iii) Find the normal force exerted by the sphere on the mass 1

iv) Explain what would happen to the mass if the tension in the string and the normal force were equal. 2

c) The equation $x^3+px^2+qx+r=0$ has one root equal to the sum of the other two. Show that $p^3-4pq+8r=0$ 3

$\alpha\beta\chi\delta\varepsilon\phi\varphi\gamma\eta\iota\kappa\lambda\mu\nu o\pi\varpi\theta\vartheta\rho\sigma\varsigma\tau\upsilon\omega\xi\psi$

S.C.H.S Extn 2
Trial HSC
2007
Solutions

1/16

1 a) i) $\sin A\cos B - \sin B\cos A + \sin A\cos B + \sin B\cos A$ ✓
$= 2\sin A\cos B$ Q.E.D.

ii) $\frac{1}{2}\int(\sin 2x + \sin 4x)\,dx$
$= -\frac{\cos 2x}{4} - \frac{\cos 4x}{8} + c$ ✓

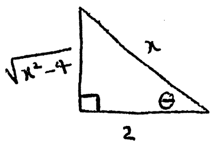
b) let $u = x^2+7$
$\frac{du}{dx} = 2x$
$\frac{du}{2x} = dx$
$\int x\sqrt{u}\,\frac{du}{2x}$ ✓
$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}\times 2} + c$
$= \frac{\sqrt{(x^2+7)^3}}{3} + c$ ✓

c) $\int \frac{dx}{\sqrt{x^2-4x+4+4}}$
$= \int \frac{dx}{\sqrt{(x-2)^2+4}}$ ✓
$= \ln\left(x-2+\sqrt{(x-2)^2+4}\right) + c$ ✓

d) i) $4x^2+3x+33 = (x+1)(Ax+B) + (x^2+16)C$
when $x=-1$, $34 = 17C$
$C = 2$ ✓
" $x = 0$, $33 = B + 16C$
$B = 1$ ✓
$4x^2 = Ax^2 + Cx^2$
$A = 2$ ✓

ii) $\int\left(\frac{2x+1}{x^2+16} + \frac{2}{x+1}\right)dx$
$= \int\left(\frac{2x}{x^2+16} + \frac{1}{x^2+16} + \frac{2}{x+1}\right)dx$
$= \ln(x^2+16) + \frac{1}{4}\tan^{-1}\frac{x}{4} + 2\ln(x+1) + c$ ✓ ✓ ✓

e) let $x = 2\sec\theta$ ✓
$\frac{dx}{d\theta} = -2(\cos\theta)^{-2}\times -\sin\theta$
$= 2\sec\theta\tan\theta$
$dx = 2\sec\theta\tan\theta\,.\,d\theta$



$\int\frac{8\sec\theta\tan\theta\,.\,d\theta}{4\sec^2\theta\sqrt{4\sec^2\theta-4}} = \int\frac{2\tan\theta\;d\theta}{\sec\theta\sqrt{4\tan^2\theta}}$
$= \int\frac{d\theta}{\sec\theta}$
$= \int\cos\theta\,.\,d\theta$ ✓
$= \sin\theta + c$
$= \frac{\sqrt{x^2-4}}{x} + c$ ✓

2 a) $6+8i-3i+4$
$= 10+5i$ ✓

b) i) $\frac{1-3i}{1+2i}\times\frac{1-2i}{1-2i}$
$= \frac{1-2i-3i-6}{1+4}$
$= \frac{-5-5i}{5}$
$= -1-i$ ✓

$r = \sqrt{1^2+1^2}$ $\theta = \tan^{-1}\left(\frac{-1}{-1}\right)$
$= \sqrt{2}$ ✓ $= -(\pi - \frac{\pi}{4})$
$= -\frac{3\pi}{4}$
$\therefore \sqrt{2}\,\text{cis}\left(-\frac{3\pi}{4}\right)$ ✓

ii) $(\sqrt{2})^7\,\text{cis}\left(-\frac{3\pi}{4}\times 7\right)$
$= 8\sqrt{2}\,\text{cis}\left(-\frac{21\pi}{4}\right)$
$= 8\sqrt{2}\,\text{cis}\,\frac{3\pi}{4}$ ✓

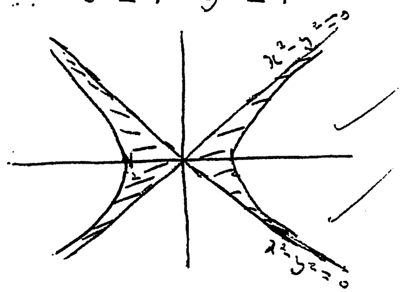
c) i) $(a+bi)^2 = -15-8i$
$a^2-b^2 = -15$ $2ab = -8$
$a = -\frac{4}{b}$ ✓
$\frac{16}{b^2} - b^2 + 15 = 0$
$16 - b^4 + 15b^2 = 0$
$b^4 - 15b^2 - 16 = 0$
$(b^2-16)(b^2+1) = 0$
$b = \pm 4$ $a = \mp 1$
$\therefore$ square roots are ✓
$\pm(1-4i)$

ii) $z = \frac{-2i+3 \pm\sqrt{(2i-3)^2 - 4(5-i)}}{2}$ ✓
$= \frac{-2i+3\pm\sqrt{-4-12i+9-20+4i}}{2}$ $= \frac{-2i+3\pm\sqrt{-15-8i}}{2}$ $= \frac{-2i+3\pm(1-4i)}{2} = \frac{4-6i}{2}$ or $\frac{2+2i}{2}$ ✓

$2-3i$
or $1+i$

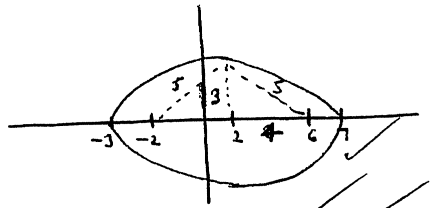
4/16.

d) $\text{Re}\{(x+iy)^2\} = \text{Re}(x^2+2ixy-y^2)$
$= x^2-y^2$ ✓
$\therefore\ 0 \le x^2-y^2 \le 4$



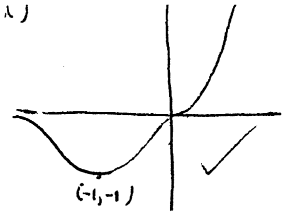
✓ ✓

e)

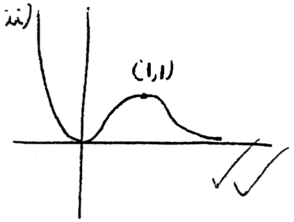


centre is $(2,0)$ $2a = 10$ ✓ $2b = 6$ ✓

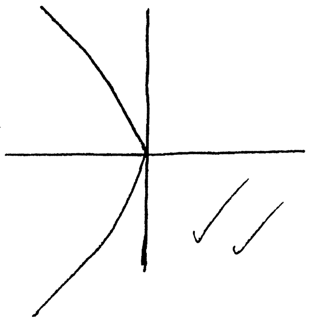
3 a) i)



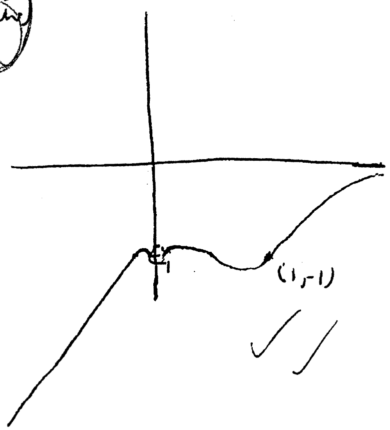
ii)



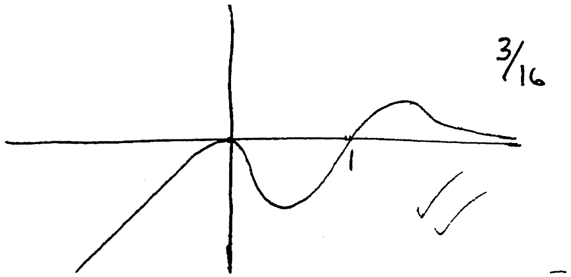
iii) $y = \pm\sqrt{f(x)}$



iv)



v)



3/16

b) i) $X\hat{Y}S = 90°$ (angle in a semi-circle)

$\hat{Q} = X\hat{Y}S$ (exterior angle of a cyclic quadrilateral equals interior opposite angle)

$= 90°$

ii) $R\hat{Y}Q = R\hat{X}Q$ (angles in same segment)

$P\hat{Y}S = P\hat{X}S$ (" " " ")

$P\hat{X}Q = P\hat{X}S + S\hat{X}R + R\hat{X}Q = 180°$ (straight angle)

$S\hat{Y}R = P\hat{Y}S + P\hat{Y}Q + R\hat{Y}Q = 180°$ (" ")

$\therefore P\hat{Y}Q = S\hat{X}R$ Q.E.D.

4/16

Question Four

(a) $\dfrac{y^2}{16} - \dfrac{x^2}{9} = 1$

(i) $e^2 = \dfrac{b^2+a^2}{b^2} = \dfrac{16+9}{16} = \dfrac{25}{16}$

$\therefore e = \dfrac{5}{4}$

(ii) Foci $(0, \pm be)$

$(0, \pm 5)$

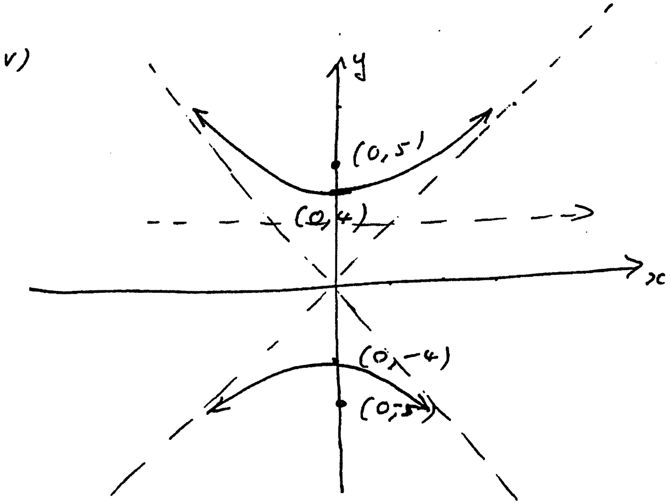
(iii) Directrices $y = \pm \dfrac{b}{e}$

$y = \pm \dfrac{4}{\frac{5}{4}} = \pm \dfrac{16}{5}$

(iv) Asymptotes $y = \pm \dfrac{b}{a} x$

$y = \pm \dfrac{4}{3} x.$

(v)



Question Four

(b) (i) $\frac{x^2}{4} - \frac{y^2}{5} = 1$ $P(2\sec\theta, \sqrt{5}\tan\theta)$

1 $\frac{(2\sec\theta)^2}{4} - \frac{(\sqrt{5}\tan\theta)^2}{5} = \frac{4\sec^2\theta}{4} - \frac{5\tan^2\theta}{5} = 1$

$\therefore$ P lies on the hyperbola $\frac{x^2}{4} - \frac{y^2}{5} = 1$

(ii) $x = 2\sec\theta$ $y = \sqrt{5}\tan\theta$

$\frac{dx}{d\theta} = 2\sec\theta\tan\theta.$ $\frac{dy}{d\theta} = \sqrt{5}\sec^2\theta$

1 $\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \frac{\sqrt{5}\sec\theta}{2\tan\theta}.$

Eqn of tangent $y - \sqrt{5}\tan\theta = \frac{\sqrt{5}\sec\theta}{2\tan\theta}(x - 2\sec\theta).$

1 $2y\tan\theta - 2\sqrt{5}\tan^2\theta = \sqrt{5}x\sec\theta - 2\sqrt{5}\sec^2\theta$

$\sqrt{5}x\sec\theta - 2y\tan\theta = 2\sqrt{5}.$

3 1 $\frac{x\sec\theta}{2} - \frac{y\tan\theta}{\sqrt{5}} = 1$

(iii) Asymptotes of hyperbola $y = \pm\frac{b}{a}x = \pm\frac{\sqrt{5}}{2}x.$

Co-ords of A. where $y = \frac{\sqrt{5}}{2}x$ cuts tangent.

$\frac{x\sec\theta}{2} - \frac{\sqrt{5}}{2}x \cdot \frac{\tan\theta}{\sqrt{5}} = 1.$

$x(\sec\theta - \tan\theta) = 2 \Rightarrow x = \frac{2}{\sec\theta - \tan\theta}$

$\Rightarrow x = \frac{2}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \Rightarrow x = 2(\sec\theta + \tan\theta)$

$\therefore y = \sqrt{5}(\sec\theta + \tan\theta)$

$A(2(\sec\theta + \tan\theta), \sqrt{5}(\sec\theta + \tan\theta))$

2

Co-ords of B where $y = -\frac{\sqrt{5}}{2}x.$

$\therefore \frac{x\sec\theta}{2} + \frac{\sqrt{5}}{2}x\frac{\tan\theta}{\sqrt{5}} = 1.$

4 $x(\sec\theta + \tan\theta) = 2$

$\Rightarrow x = \frac{2}{\sec\theta + \tan\theta} \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$

$\therefore x = 2(\sec\theta - \tan\theta)$

$y = -\frac{\sqrt{5}}{2} \cdot 2(\sec\theta - \tan\theta)$

2 $y = \sqrt{5}(\tan\theta - \sec\theta)$

$B(2(\sec\theta - \tan\theta), \sqrt{5}(\tan\theta - \sec\theta))$

(iv) $A(2(\sec\theta + \tan\theta), \sqrt{5}(\sec\theta + \tan\theta))$

$B(2(\sec\theta - \tan\theta), \sqrt{5}(\tan\theta - \sec\theta))$

Mid-pt $x = \frac{2\sec\theta + 2\tan\theta + 2\sec\theta - 2\tan\theta}{2}$

$x = 2\sec\theta$

2 $y = \frac{\sqrt{5}\sec\theta + \sqrt{5}\tan\theta + \sqrt{5}\tan\theta - \sqrt{5}\sec\theta}{2}$

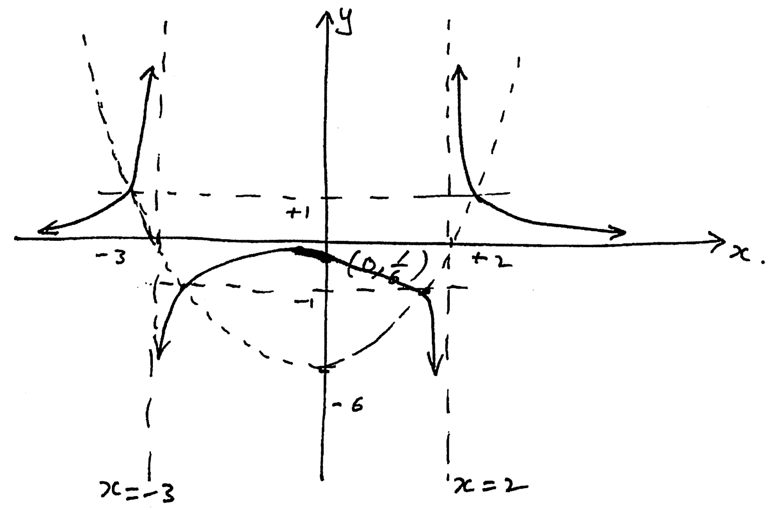
$y = \sqrt{5}\tan\theta$

which is $P(2\sec\theta, \sqrt{5}\tan\theta)$

$\therefore$ P is mid-pt of AB

Question Five

ⓐ $y = \dfrac{1}{x^2 + x - 6} = \dfrac{1}{(x+3)(x-2)}.$



3

ⓑ $x^2 + y^2 + xy - 4 = 0.$

Require tangent at (0, 2)

$2x + 2y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y + x) + (2x + y) = 0$

$\therefore \frac{dy}{dx} = \frac{-(2x+y)}{2y+x}$

3

at (0, 2) $\frac{dy}{dx} = -\frac{(2)}{4} = -\frac{1}{2}$

Eqn of tangent $y - 2 = -\frac{1}{2}(x - 0)$

$y - 2 = -\frac{1}{2}x.$

$2y - 4 = -x$

$x + 2y - 4 = 0$

Question Five

ⓒ $P(x) = x^4 - 4x^3 + 11x^2 - 14x + 10.$

roots $a+ib$ and $a+2ib$

by property of conjugates $a-ib$ and $a-2ib$ are also roots

Product of roots $(a+ib)(a-ib)(a+2ib)(a-2ib)$

$= (a^2+b^2)(a^2+4b^2)$

Sum of roots

$a+ib + a-ib + a+2ib + a-2ib = 4$

$\therefore 4a = 4 \Rightarrow a = 1$

Now $(1+b^2)(1+4b^2) = 10$

3

$1 + 5b^2 + 4b^4 = 10.$

$4b^4 + 5b^2 - 9 = 0$ Put $A = b^2$

$4A^2 + 5A - 9 = 0$

$(4A+9)(A-1) = 0$

$\therefore A = -\frac{4}{9}$ $A = 1$

$A \neq -\frac{4}{9}$

as $A = b^2$

$\therefore b^2 = 1$

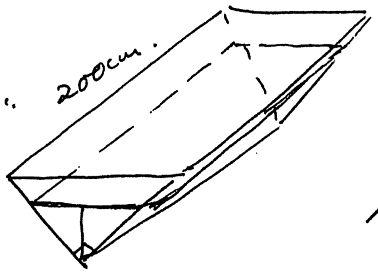
$\therefore b = \pm 1$

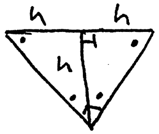
$\therefore$ roots are $\left.\begin{matrix} 1 \pm i \\ 1 \pm 2i \end{matrix}\right\}$

Question Five

(a)

$\frac{dV}{dt} = 12cm^3/sec.$





Area of $\Delta = \frac{1}{2}.b.h$
$= \frac{1}{2}.2h \times h = \underline{h^2}$

∴ Volume of water when water level = h

$V = 200 \times h^2\ cm^3$ $\qquad \frac{dV}{dh} = 400h.$

Area of Upper Surface $A = 200 \times 2h = 400h\ cm^2.$

Require $\frac{dA}{dt}$ $\qquad \frac{dA}{dt} = \frac{dA}{dh} \cdot \frac{dh}{dV} \cdot \frac{dV}{dt}$ $\qquad \frac{dA}{dh} = 400$

when $h = 12.$

$\frac{dA}{dt} = 400 \times \frac{1}{400h} \times 12cm^3/se$

$h = 12$

6 $\quad ∴ \frac{dA}{dt} = \underline{1cm^2/sec}$

Question Six

(a) $x^3 + 6x^2 + 5x + 5 = 0.$

has roots $\alpha, \beta, \gamma.$

$∴ \alpha + \beta + \gamma = -6 \quad \alpha\beta + \beta\gamma + \gamma\alpha = 5 \quad \alpha\beta\gamma = -5.$

(i) require eqn with roots $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}.$

$∴ X = \frac{1}{x} \Rightarrow x = \frac{1}{X}$

$∴ P(X) = (\frac{1}{X})^3 + 6(\frac{1}{X})^2 + 5(\frac{1}{X}) + 5 = 0$

2 $\quad \frac{1}{X^3} + \frac{6}{X^2} + \frac{5}{X} + 5 = 0$

$∴ P(x) = 5x^3 + 5x^2 + 6x + 1 = 0$

(ii) require eqn with roots $\frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\alpha\gamma}$

i.e. $\frac{\gamma}{\alpha\beta\gamma}, \frac{\alpha}{\alpha\beta\gamma}, \frac{\beta}{\alpha\beta\gamma}.$

∴ eqn with roots. $\frac{\alpha}{-5}, \frac{\beta}{-5}, \frac{\gamma}{-5}.$

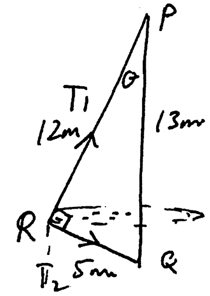
2 $\quad ∴ \quad X = \frac{x}{-5} \Rightarrow x = -5X.$

$∴ P(X) = (-5X)^3 + 6(-5X)^2 + 5(-5X) + 5 = 0$

$P(X) = -125X^3 + 150X^2 - 25X + 5 = 0.$

$∴ P(x) = 25x^3 - 30x^2 + 5x - 1 = 0$

(b)



5

Vertically:

$T_1 \cos\theta = T_2 \cos(90 - \theta) + mg.$

$T_1 \cos\theta = T_2 \sin\theta + mg.$

$T_1 \times \frac{12}{13} = T_1 \times \frac{5}{13} + 40.$

$12T_1 = 5T_1 + 40 \times 13$

$7T_1 = 40 \times 13$

$T_1 = \frac{40 \times 13}{7}$ Newtons.

$\sin\theta = \frac{r}{12}$

$r = 12 \times \frac{5}{13}$

$r = \frac{60}{13}$

Horizontally: $T_1 \sin\theta + T_2 \cos\theta = m\frac{v^2}{r}$

$T_1(\frac{5}{13} + \frac{12}{13}) = 4 \times \frac{v^2}{\frac{60}{13}} \Rightarrow \frac{40 \times 13}{7}(\frac{17}{13}) = \frac{4 \times 13}{60}v^2.$

$∴ v^2 = \frac{600 \times 17}{7 \times 13} \Rightarrow v \doteqdot 10.6m/sec$

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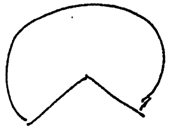
Question Six

(c)



(i) Linear relationship $r = mh + b$.

$h = 0$ when $r = 2$ $\therefore b = 2$.

$h = 20$ when $r = 8$.

$\therefore 8 = 20m + 2$

$6 = 20m \Rightarrow m = \frac{3}{10}$.

$\therefore r = \frac{3}{10}h + 2$

Linear relationship $\theta = nh + c$

$h = 0$ $\theta = \frac{7\pi}{4}$

$\therefore c = \frac{7\pi}{4}$.

3

$h = 20$ $\theta = \frac{5\pi}{4}$ $\therefore \frac{5\pi}{4} = 20n + \frac{7\pi}{4}$

$\therefore 20n = -\frac{\pi}{2}$

$n = -\frac{\pi}{40}$.

$\therefore \theta = -\frac{\pi}{40}h + \frac{7\pi}{4}$

$\theta = \frac{\pi}{4}\left(7 - \frac{h}{10}\right)$.

(ii) Area of a slice $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\cdot\left(\frac{3}{10}h + 2\right)^2\cdot\frac{\pi}{4}\left(7 - \frac{h}{10}\right)$

Volume of a slice $\delta V = \frac{\pi}{8}\left(\frac{3h}{10} + 2\right)^2\left(7 - \frac{h}{10}\right).\delta h$

Volume of Solid $V \doteq \sum_0^{20} \frac{\pi}{8}\left(\frac{3h+20}{10}\right)^2\left(7 - \frac{h}{10}\right).\delta h$.

$\lim \delta h \to 0$

3 by defn. $V = \frac{\pi}{8}\int_0^{20}\left(\frac{3h+20}{10}\right)^2\left(\frac{70-h}{10}\right)dh$.

$V = \frac{\pi}{8000}\int_0^{20}(9h^2 + 120h + 400)(70 - h).dh$.

$V = \frac{\pi}{8000}\int_0^{20}(630h^2 + 8400h + 28000 - 9h^3 - 120h^2 - 400h)dh$

$V = \frac{\pi}{8000}\left[210h^3 + 4200h^2 + 28000h - \frac{9}{4}h^4 - 40h^3 - 200h^2\right]$

$V = \frac{\pi}{8000}\left[8000(210 + 210 + 70 - 45 - 40 - 10)\right] = 395\pi \text{ cm}^3$

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Question Seven

a) $P(x) = 3x^3 - 11x^2 + 8x + 4$

$P'(x) = 9x^2 - 22x + 8$

put $P'(x) = 0$

$9x^2 - 22x + 8 = 0$

$(9x - 4)(x - 2) = 0$ 1

$x = \frac{4}{9}$ or $x = 2$

$P(2) = 0$ $\therefore$ $x = 2$ is double root 1

$P(x) = (x-2)^2(3x+1)$ 1

(3)

b) i) $(\cos\theta + i\sin\theta)^5 = c^5 + 5c^4 i s - 10c^3s^2 - 10 i c^2 s^3 + 5cs^4 + i s^5$

Now $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

Equate imaginary 1

$\sin 5\theta = 5c^4 s - 10c^2s^3 + s^5$

$= 5(1-s^2)^2 s - 10(1-s^2)s^3 + s^5$

$= 5(1 - 2s^2 + s^4)s - 10s^3 + 10s^5 + s^5$ (3)

$= 5s - 10s^3 + 5s^5 - 10s^3 + 10s^5 + s^5$

$\sin 5\theta = 16\sin^5\theta - 20s^3 + 5s$ 1

ii) $\sin x = 16\sin^5 x - 20\sin^3 x + 5\sin x$

$0 = 16\sin^5 x - 20\sin^3 x + 4\sin x$

$0 = 4\sin x(4\sin^4 x - 5\sin^2 x + 1)$

$0 = 4\sin x(4\sin^2 x - 1)(\sin^2 x - 1)$ 1

$4\sin x = 0$, $\sin^2 x = \frac{1}{4}$ $\sin^2 x = 1$ (3)

$\sin x = 0$ $\sin x = \pm\frac{1}{2}$ $\sin x = \pm 1$ 1

$x = 0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$ 1

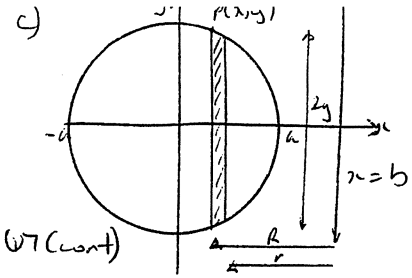
0, 180, 360, 30°, 150°, 210° 330°, 90°, 270°

(9 Solns)

	A	B	C	D	E	F	G	H
Q7	2	1	6	8	7	4	5	3
Q8.								

Notes
Individual parts mark circled
Total underlined and circled
N.A or missing question paper check and initialled

c)



Q7 (cont)

i) $R = b - x$

$r = b - x - \Delta x$

$y^2 = a^2 - x^2$

$y = \sqrt{a^2 - x^2}$ 1 (all correct)

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$V_{shell} = \pi(R^2 - r^2)\, 2y$

$= \pi(R+r)(R-r)\, 2y$ (3)

$= \pi(2b - 2x - \Delta x)(b - x - b + x + \Delta x)2y$

$= \pi(2b - 2x - \Delta x)\Delta x\, 2y$

$= \pi(2b\Delta x - 2x\Delta x - (\Delta x)^2)\, 2y$

$= \pi(2b - 2x)\, 2y\Delta x\, [(\Delta x^2)$ v small$]$

$= 4\pi(b - x)\Delta x y$

$= 4\pi(b-x)\Delta x\sqrt{a^2 - x^2}$

2

ii) $V_{solid} = \lim_{\Delta x \to 0} 4\pi(b-x)\sqrt{a^2 - x^2}\,\Delta x$

$= 4\pi\int_{-a}^{a}(b - x)\sqrt{a^2 - x^2}\, dx$

Note if answer to part i) is $4\pi x\sqrt{a^2 - x^2}\,dx$ or $4\pi b\sqrt{a^2-x^2}\,dx$ then a max of 1 mark for ii)

$= 4\pi b\int_{-a}^{a}\sqrt{a^2 - x^2}\, dx - 4\pi\int_{-a}^{a} x\sqrt{a^2 - x^2}\, dx$ 1

(3)

$= 4\pi b\int_{-a}^{a}\sqrt{a^2 - x^2}\, dx - 0$ $[\int_{-a}^{a} x\, a^2 - x^2\, dx$ odd$]$ 1

$= 4\pi b\left(\frac{\pi a^2}{2}\right)$ (area of a semi circle 1)

$= 2\pi^2 a^2 b$

16692093 c) i does not follow through (1 mark) to ii) 16691747 N. Paper

19471357 Changed question c) from shells to slices

Question Eight

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a)

i) 1) When $n = 1$

LHS $= \cos(x + \pi)$

$= \cos x\cos\pi - \sin x\sin\pi$

$= -\cos x$

RHS $= -\cos x$ true for $n = 1$ 1

2) Assume true for $n = k$

$\cos(x + k\pi) = (-1)^k\cos x$

3) Prove true for $n = k+1$

ie $\cos[(x + (k+1)\pi] = (-1)^{k+1}\cos x$

LHS $= \cos[(x + k\pi) + \pi]$

$= \cos(x + k\pi)\cos\pi - \sin(x + k\pi)\sin\pi$ 1

$= (-1)^k\cos^* x\,(-1)^1 - (0)$ 1 *from assumpt (only if it follows from full line above)

$= (-1)^{k+1}\cos x$

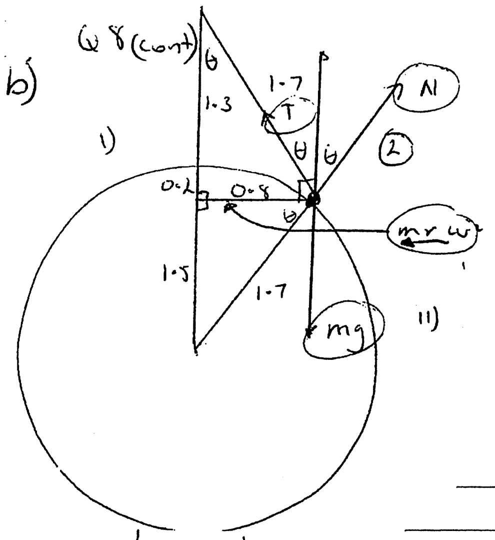
$=$ RHS

4) Hence if true for $n = k$, true for $n = k+1$, true for $n = 1$, hence true for $n = 1+1 = 2$ and so on for all postive integers 1 (4)

Notes

- All proofs should flow, each step should lead to the step below.
- at step 3 write out what you aim to prove and work on one side or two sides independantly
- Justify each line where needed

b)



Label T, N, mrw², N for two marks

b cont

b) ii) $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$, $m = 6$ kg, $g = 10\ \text{ms}^{-2}$, $w = 2\ \text{rad s}^{-1}$

Vertically

$T\cos\theta + N\cos\theta = mg$ ⇐ 1

$\frac{15T}{17} + \frac{15N}{17} = 60$

$15T + 15N = 1020$

$N = 68 - T$ (A)

Horizontally

$T\sin\theta - N\sin\theta = mrw^2$

$\frac{8T}{17} - \frac{8N}{17} = 6(0\cdot8)(4)$ ⇐ 1   can be CFPA if $(90° - \theta)$ used

$8T - 8N = 326\cdot4$

$T - N = 40\cdot8$

$T - (68 - T) = 40\cdot8$

$2T = 108\cdot8$

$T = 54\cdot4$ Newtons 1

iii) $N = 68 - 54\cdot4$ [CAN BE CFPA]

$= 13\cdot6$ Newtons 1

iv) If $T = N$ [NO CARRY THROUGH ERROR]

then $T\sin\theta - T\sin\theta = mrw^2$

∴ w must equal zero then

If $w = 0$ mass is not moving (2 marks)

2

Q8 (cont)

Method 1

c) roots are $\alpha, \beta, \alpha+\beta$

(1) Sum of roots $2\alpha + 2\beta = -p$

$\alpha + \beta = -\frac{p}{2}$ 1

Now $\alpha+\beta$ is a root ∴ $-\frac{p}{2}$ is a root

subst $x = -\frac{p}{2}$ in $x^3 + px^2 + qx + r = 0$

$\left(-\frac{p}{2}\right)^3 + p\left(-\frac{p}{2}\right)^2 + q\left(-\frac{p}{2}\right) + r = 0$ 1

$-\frac{p^3}{8} + \frac{p^3}{4} + \frac{pq}{2} + r = 0$

$-p^3 + 2p^3 + 4pq + 8r = 0$

$p^3 + 4pq + 8r = 0$ 1

(3)

Method 2

c) $\alpha = \beta + \gamma$

∴ roots are $(\beta+\gamma), \beta, \gamma$

Sum $2\beta + 2\gamma = -p \Rightarrow -2(\beta+\gamma) = p$

Product two at a time

$(\beta+\gamma)\beta + (\beta+\gamma)\gamma + \beta\gamma = q$

$\beta^2 + \gamma\beta + \beta\gamma + \gamma^2 + \beta\gamma = q$

$(\beta+\gamma)^2 + \alpha\beta = q$

Product $\beta\gamma(\beta+\gamma) = -r$

} 2

subst in $p^3 - 4pq + 8r = 0$

$\text{LHS} = [-2(\beta+\gamma)]^3 - 4[-2(\beta+\gamma)][(\beta+\gamma)^2 + \alpha\beta] + 8[\beta\gamma(\beta+\gamma)]$

$= -8(\beta+\gamma)^3 + 8[(\beta+\gamma)^3 + (\beta+\gamma)\alpha\beta] - 8[\beta\gamma(\beta+\gamma)]$

$= -8(\beta+\gamma)[(\beta+\gamma)^2 - (\beta+\gamma)^2 - \alpha\beta + \alpha\beta]$

$= 0$ 1

A	B	C	D	E	F	G	H
2·	8	5	1	7·	3	6	4